

The large- N_c limit and the behavior of $g_A^{(0)}$ and g_A Atsushi Hosaka⁽¹⁾ and Niels R. Walet⁽²⁾⁽¹⁾*Numazu College of Technology, 3600 Ooka, Numazu, Shizuoka, 410 Japan*⁽²⁾*Institut für theoretische Physik III, Universität Erlangen-Nürnberg,
D-91058 Erlangen, Germany***Abstract**

We investigate the isoscalar and isovector components of the axial vector coupling constants, $g_A^{(0)}$ and g_A using algebraic models that lead to the correct symmetries of large- N_c QCD. Results obtained previously in various chiral models are interpreted from this algebraic point of view. The results of the Skyrme model and the valence quark model are explained by simple realizations of the algebra.

1 Introduction

In recent studies of large- N_c baryons and mesons [1, 2, 3, 4], many results derived originally in the Skyrme model, the chiral bag model and the chiral quark soliton model have been obtained using algebraic methods, where the algebra is inferred from the behavior of large- N_c QCD [5]. The algebraic method does not depend on the details of dynamics, and can provide a clear understanding of the results obtained in specific models. Furthermore, it provides a method for systematic calculation of higher order terms in the $1/N_c$ expansion for meson-baryon coupling constants, magnetic moments and other quantities [1, 2, 3, 4].

A different but related algebraic method was developed by Amado and collaborators for finite N_c corrections to the Skyrme model, borrowing ideas from the interacting boson model in nuclear physics [6, 7]. From the perspective of the modern work on large- N_c QCD it has now become clear that the group considered by them is the minimal one consistent with the large- N_c behavior of QCD. This immediately raises the question whether other realizations of

the algebra exist, that have the same limit for $N_c \rightarrow \infty$, but lead to different predictions for finite N_c . In this paper, we discuss this question looking at the nucleon matrix elements of σ_i and $\sigma_i \tau^a$, which are related to the axial vector coupling constants $g_A^{(0)}$ and g_A .

Our motivation is twofold:

- (1) The fact that those quantities were calculated successfully in several different chiral models [8, 9, 10] strongly suggests that there should exist a feature common to those models, which is based on the underlying algebraic structure. Our main purpose is therefore to construct an algebraic model which is able to explain those results.
- (2) In the algebraic method, Skyrme model results have been interpreted to be the limit of $N_c \rightarrow \infty$ [11, 12]. Specifically, this has been explicitly shown using the so called quark representation, the symmetric representation with Young tableau $[N_c]$ of the $SU(4)$ group generated by spin and isospin. A seeming advantage of this representation is that it produces not only the large- N_c limit correctly but also the quark model results at finite N_c . It turns out, however, that this is not always the case. For a quantity such as the nucleon spin $g_A^{(0)}$ the Skyrme model result, which is essentially zero [13], is not reproduced in the quark representation.

Therefore, we construct an appropriate parameterization which reproduces both the Skyrme and the quark model limits for $g_A^{(0)}$ and g_A . This requirement leads to a study of models with a dynamical symmetry group that is large enough to have $SU(4)$ as a subgroup. In order to get interesting models we let ourselves be guided by some aspects of effective models of QCD, that might be particularly relevant for the physical questions considered here. Of course a dynamical symmetry requires that we associate some dynamics with the models we consider. In the present note we shall concentrate on the states, and will not discuss what Hamiltonians lead to such a state. Fortunately, such Hamiltonians do exist. They will be the subject of a separate study.

2 Algebra for large- N_c QCD

In the following two sections, we repeat some of the essential details of the large- N_c algebra as first set out in the paper by Gervais and Sakita [5]. For that purpose consider the pion-nucleon scattering amplitude. To leading order, the relevant Born terms are given by

$$\mathcal{M} \sim N_c \sum_Y \left\{ \frac{\mathcal{V}_{XY}^\beta \mathcal{V}_{YZ}^\alpha}{\omega + M_Z - M_Y} - \frac{\mathcal{V}_{XY}^\alpha \mathcal{V}_{YZ}^\beta}{\omega - M_X + M_Y} \right\}, \quad (1)$$

Here $\sqrt{N_c}\mathcal{V}_{XY}^\alpha$ are the Yukawa vertices with the $\sqrt{N_c}$ dependence factored out, and $M_{X\dots}$ are the masses of the baryons. In these quantities, subscripts X, Y, \dots specify the quantum numbers for the baryons which are in the fundamental representations of the spin and isospin symmetries, $X \equiv (J(\text{spin}), I(\text{isospin}))$ etc, while α, β stand for the quantum numbers of the pions. The latter label the adjoint representations of the same spin-isospin group, since the pion carries isospin one and it couples with the nucleon through the P-wave. From unitarity and also from the Witten's large- N_c counting rule [14], the scattering amplitude is bounded from below for arbitrary N_c . Thus the two terms in (1) must cancel each other in the limit $N_c \rightarrow \infty$. This is the primary consistency condition for large- N_c QCD [1, 3]. It is satisfied if the mass differences of the tower of states built upon the nucleon go to zero in this limit and the following relation holds,

$$\sum_Y \left(\mathcal{V}_{XY}^\beta \mathcal{V}_{YZ}^\alpha - \mathcal{V}_{XY}^\alpha \mathcal{V}_{YZ}^\beta \right) \xrightarrow{N_c \rightarrow \infty} 0. \quad (2)$$

Here the left hand side must be suppressed as proportional to $1/N_c$. Equation (2) is now regarded as a matrix equation for \mathcal{V}^α whose components are \mathcal{V}_{XY}^α .

In the limit $N_c \rightarrow \infty$, combining the spin and isospin algebra for light flavors, $K = \text{SU}(2) \times \text{SU}(2)$, with that generated by (2), we obtain the following commutation relations:

$$\begin{aligned} [\mathcal{J}^a, \mathcal{J}^b] &= if^{abc} \mathcal{J}^c, \\ [\mathcal{J}^a, \mathcal{V}^\alpha] &= iD(a)_\beta^\alpha \mathcal{V}^\beta, \\ [\mathcal{V}^\alpha, \mathcal{V}^\beta] &= 0. \end{aligned} \quad (3)$$

Here $\mathcal{J}^a \in K$ and f^{abc} are the generators and the structure constants for the group K . $D(a)_\beta^\alpha$ are the matrices of the generators \mathcal{J}^a in the basis of α : $D(a)_\beta^\alpha \equiv \langle \alpha | J^a | \beta \rangle$. Equations (3) shows that \mathcal{J}^a and \mathcal{V}^α form the non-compact group algebra $G = K \times T$, the semi-direct product of K by the Abelian group T generated by \mathcal{V}^α . Because of this non-compact nature of the group, the baryon states form an infinite tower of states with spin equal to isospin $(I, J) = (\frac{1}{2}, \frac{1}{2}), (\frac{3}{2}, \frac{3}{2}),$ etc.

3 Representations of the large- N_c group

In Ref. [5] the construction of representations for the large- N_c group $G = K \times T$ is performed by the method of induced representations. Equivalently one can use the technique of group contractions starting from the compact group $\text{SU}(4)$. Here we see naturally how the extrapolation to infinite N_c is performed.

The first step in the argument is based on the fact that the number of generators of $K \times T$, with $K = \text{SU}(p) \times \text{SU}(q)$ and the T a direct product of the adjoint representations of $\text{SU}(p)$ and $\text{SU}(q)$,

$$N = (p^2 - 1) + (q^2 - 1) + (p^2 - 1)(q^2 - 1) = (pq)^2 - 1, \quad (4)$$

is the same as the number of generators of the $\text{SU}(pq)$ group. Let us look at the $\text{SU}(p) \times \text{SU}(q)$ subgroup of $\text{SU}(pq)$. Let us denote the generators of the subgroup by \mathcal{J} , and the remaining generators by \mathcal{W} . The algebra now takes the form

$$\begin{aligned} [\mathcal{J}^a, \mathcal{J}^b] &= if^{abc} \mathcal{J}^c, \\ [\mathcal{J}^a, \mathcal{W}^\alpha] &= iD(a)_\beta^\alpha \mathcal{W}^\beta, \\ [\mathcal{W}^\alpha, \mathcal{W}^\beta] &= i \left(g_1^{\alpha\beta\gamma} \mathcal{W}^\gamma + g_2^{\alpha\beta c} \mathcal{J}^c \right). \end{aligned} \quad (5)$$

Here f , g_1 , g_2 are the structure constants. For the case of $p = q = 2$, the constants g_1 vanishes and the last commutators of (5) can be written as

$$[\mathcal{W}^{ia}, \mathcal{W}^{jb}] = \frac{i}{4} \epsilon_{ijk} \delta_{ab} J^k + \frac{i}{4} \epsilon_{abc} \delta_{ij} I^c, \quad (6)$$

where J^i and I^a are the generators of the spin and isospin group.

Consider now the N -dimensional symmetric representation of $\text{SU}(4)$ and suppose that the matrix elements of \mathcal{W} are of order N . The group contraction $\text{SU}(4) \rightarrow G = K \times T$ can be performed by first replacing \mathcal{W}^α by $\mathcal{V}^\alpha \equiv \mathcal{W}^\alpha/N$. In the limit $N \rightarrow \infty$ the algebra (5) written in terms of \mathcal{V} then reduces to the algebra (3). The index N can then be identified with the number of colors in what has been called the “quark representation” (which refers to valence quarks) as shown in the following.

Consider the wave function of a baryon. It can be written as a direct product of orbital, spin, isospin (flavor) and color parts. Since the color part is totally antisymmetric (it has Young tableau $[(1)^{N_c}]$) the rest of the wave function must be totally symmetric, with Young tableau $[N_c]$. For ground state baryons, quarks are assumed to be in the lowest S state, which gives a symmetric orbital wave function, and thus the spin and isospin part must be symmetric as well. The symmetric representation of $\text{SU}(4) \supset \text{SU}(2) \times \text{SU}(2)$ is specified by an index N , which must now be identified with the number of colors N_c .

Let us introduce operators which generate the fundamental representations for spin and isospin group, $\alpha_{\mu\nu}$. For instance, $\alpha_{u\downarrow}$ annihilates a spin down up quark. The symmetric N_c

representation of the $SU(4)$ group is then generated by the defensive hedgehog state [6, 7] (the name defensive hedgehog state derives from the Skyrme model, where this state corresponds to a pion field that point radially outwards, as on a hedgehog defending itself)

$$|N_c; h_q\rangle = \sqrt{\frac{1}{N_c!}} \left[\frac{1}{\sqrt{2}} (\alpha_{u\downarrow}^\dagger - \alpha_{d\uparrow}^\dagger) \right]^{N_c} |0\rangle, \quad (7)$$

where on the left hand side the subscript q indicates that the state is the quark representation. The operator in square brackets is exactly what is obtained by coupling the spin and isospin to zero. This coupling to “grand-spin” $K = 0$ is typical of a hedgehog state. The state (7) breaks rotational and iso-rotational symmetries. This is possible since all states obtained by applying an isospin rotation operator $R(A)$ (where A is the two-by-two unitary matrix specifying the transformation) on the defensive hedgehog state (7) are degenerate,

$$\begin{aligned} |N_c; h_q[A]\rangle &\equiv R(A)|N_c; h\rangle \\ &= \sqrt{\frac{1}{N_c!}} \left[\frac{1}{\sqrt{2}} \left(D_{uu}^{1/2}(A) \alpha_{u\downarrow}^\dagger + D_{du}^{1/2}(A) \alpha_{d\downarrow}^\dagger - D_{dd}^{1/2}(A) \alpha_{d\uparrow}^\dagger - D_{ud}^{1/2}(A) \alpha_{u\uparrow}^\dagger \right) \right]^{N_c} |0\rangle. \end{aligned} \quad (8)$$

Here $D_{\mu\nu}^{1/2}(A)$ are the D -functions of rank 1/2. The semi-classical nature in the limit $N_c \rightarrow \infty$ is reflected in the overlap function,

$$\langle N_c; h[A'] | N_c; h[A] \rangle = [\cos \theta_{AA'}]^{N_c}, \quad (9)$$

where $\theta_{AA'}$ represents symbolically the relative angle between the “orientations” A and A' . After an appropriate rescaling of the states this overlap goes to a delta function [15]

$$\langle N_c; h[A'] | N_c; h[A] \rangle = \delta(A - A'). \quad (10)$$

This sharp peaking is characteristic of a semiclassical limit.

4 Nucleon matrix elements

The nucleon state $|N\rangle$ can be projected out from the hedgehog state $|h\rangle$ by taking the appropriate combination of rotated hedgehogs [5, 12],

$$|N\rangle \propto \int d[A] D_{t,-m}^{1/2*}(A) R(A) |h\rangle. \quad (11)$$

Here the nucleon third components of spin and isospin equal m and t . The nucleon matrix elements of an observable \mathcal{O}_N can now be computed as

$$\mathcal{O}_N = \frac{\langle N | \mathcal{O} | N \rangle}{\langle N | N \rangle}, \quad (12)$$

where the denominator is needed for normalization.

Let us now calculate the two axial-vector coupling constants for the nucleon, $g_A^{(0)}$ and g_A . For $g_A^{(0)}$ we should consider the $U(1)_A$ effects seriously, but these effects will be ignored here, and we shall see how a simple construction behaves for these quantities.

The actual computation is performed most conveniently using the Euler angles α , β and γ for the rotation matrix A . The computational procedure is straightforward [15] and we just give the results here. $g_A^{(0)}$ and g_A are defined by the nucleon matrix elements of $S_3 \equiv \sum_{n=1, N_c} \sigma_i(n)$ and $T_{33} \equiv \sum_{n=1, N_c} \sigma_i(n)\tau_a(n)$. The expectation values in the $p \downarrow$ state are

$$\begin{aligned} \langle p \downarrow | S_3 | p \downarrow \rangle &= -N_c \frac{\int d[A] C_\beta^2 S^2 C^{N_c-1}}{\int d[A] C^{N_c+1}} \\ &= -\frac{N_c}{N_c} = -1. \end{aligned} \quad (13)$$

and

$$\begin{aligned} \langle p \downarrow | T_{33} | p \downarrow \rangle &= -\frac{N_c}{3} \frac{\int d[A] (C^{N_c+1} + C^{N_c-1} S^2)}{\int d[A] C^{N_c+1}} \\ &= -\frac{N_c}{3} \left(1 + \frac{2}{N_c} \right). \end{aligned} \quad (14)$$

Here we have introduced the notations $C_\beta = \cos \frac{\beta}{2}$, $C = \cos \frac{\beta}{2} \cos \frac{\alpha+\gamma}{2}$, $S = \sin \frac{\alpha+\gamma}{2}$, and $d[A] = \sin \beta d\beta d\alpha d\gamma$. The minus signs appear because the matrix elements are evaluated for the $p \downarrow$ state. The result (13), $\langle p \downarrow | S_3 | p \downarrow \rangle = -1$, as independent of N_c is trivial, since the nucleon spin here is entirely carried by the intrinsic quark spin, which is the result of the non-relativistic valence quark model. On the other hand, the result (14) depends on N_c ; it becomes (neglecting the minus sign) $5/3$ when $N_c = 3$ as in the valence quark model, while it approaches N_c in the limit $N_c \rightarrow \infty$ as corresponding to the Skyrme model result. Thus the quark representation is believed to interpolate between the quark model and the Skyrme model results when N_c goes from 3 to ∞ [12]. This is the case for g_A , but is not true for the other matrix element of the nucleon spin, $g_A^{(0)} \sim \langle p \downarrow | S_3 | p \downarrow \rangle$.

5 Other algebraic realizations

In this section we consider other algebraic realizations for the group $SU(4)$ which is contracted to the large- N_c algebra in the limit $N_c \rightarrow \infty$. By doing this, we will see a novel behavior of the

matrix elements for $g_A^{(0)}$ and g_A .

First we consider the relativistic effects which involve the lower components of the wave function with orbital angular momentum $L = 1$. Formally this can be achieved by first extending the rotational group $SU(2)$ to $SU(2)_S \times O(3)_L$ for spin (S) and orbital angular momentum (L) group, and then take its diagonal subgroup: $SU(2)_S \times O(3)_L \supset SU(2)_{J=S+L}$, where the generators of the diagonal group is the sum $\vec{J} = \vec{S} + \vec{L}$. The $SU(2)_J$ group is now combined with the isospin $SU(2)_I$ group to form the desired chain $SU(4)_q \supset SU(2)_J \times SU(2)_I$, where the subscript q on the left hand side indicates that this algebra is realized by the quarks. We shall not discuss other possible dynamical origins of orbital excitations in this note, which are discussed in great detail in Ref. [15].

The second extension is to include explicit pions, that also form a hedgehog state. Since a pion can form a grand-spin $K = 0$ state through the coupling of its unit isospin and $L = 1$ orbital angular momentum (P-wave), the relevant group is $O(3)_I \times O(3)_L$, which we once more imbed in an $SU(4)$ group. Therefore, the extended group for the hedgehog quarks and the pions would be

$$SU(4)_q \times [O(3)_I \times O(3)_L]_\pi \subset SU(4)_q \times SU(4)_\pi. \quad (15)$$

Once again, we pick up the diagonal subgroup, $SU(4)_q \times SU(4)_\pi \supset SU(4)_{q+\pi}$.

The relativistic hedgehog quarks are now written as

$$|N_c; h\rangle = \begin{pmatrix} c_1 \left[\begin{smallmatrix} \frac{1}{2} & 0 \end{smallmatrix} \right]^{J=1/2, \frac{1}{2}} \left[\begin{smallmatrix} 1 \\ 2 \end{smallmatrix} \right]^{K=0} \\ c_2 \left[\begin{smallmatrix} \frac{1}{2} & 1 \end{smallmatrix} \right]^{J=1/2, \frac{1}{2}} \left[\begin{smallmatrix} 1 \\ 2 \end{smallmatrix} \right]^{K=0} \end{pmatrix}^{N_c} |0\rangle \equiv \left[\begin{pmatrix} c_1 \\ c_2 \vec{\sigma} \cdot \hat{x} \end{pmatrix} \chi^\dagger \right]^{N_c} |0\rangle. \quad (16)$$

Here the coupling scheme is $[[S, L]^J, I]^K$ and $\chi^\dagger \equiv \left[\begin{smallmatrix} \frac{1}{2} & 0 \end{smallmatrix} \right]^{J=1/2, \frac{1}{2}} \left[\begin{smallmatrix} 1 \\ 2 \end{smallmatrix} \right]^{K=0} = \frac{1}{\sqrt{2}}(\alpha_{u\downarrow}^\dagger - \alpha_{d\uparrow}^\dagger)$. The coefficients c_1 and c_2 , $|c_1|^2 + |c_2|^2 = 1$, dictate the ratio of the upper ($L = 0$) and lower ($L = 1$) components, the precise values of which are determined dynamically. The last expression of (16) is useful in actual computation. We wrote the matrix σ instead of the algebraic operator S to exhibit the fact that it only acts on a *single* operator χ . The transformation $\vec{\sigma} \cdot \hat{x}$ acts on the spin components of the operator χ .

The matrix elements for S_3 and T_{33} are computed in a straightforward manner and the results are

$$\langle p \downarrow | S_3 | p \downarrow \rangle = - \left(|c_1|^2 - \frac{1}{3} |c_2|^2 \right) \equiv -g_A^{(0)} \quad (17)$$

and

$$\langle p \downarrow | T_{33} | p \downarrow \rangle = - \left(|c_1|^2 - \frac{1}{3} |c_2|^2 \right) \frac{N_c}{3} \left(1 + \frac{2}{N_c} \right) \equiv -g_A. \quad (18)$$

We find the same suppression factor $(|c_1|^2 - \frac{1}{3}|c_2|^2)$ for both the matrix elements. They have been included in relativistic quark model calculations such as in the bag and the chiral quark soliton models [8, 9, 10]. For the maximally relativistic case of massless quarks, it becomes 0.654 as in the MIT bag model. Accordingly, $g_A^{(0)} = 0.654$ and $g_A = 5/3 \cdot 0.654 = 1.09$ [16]. If we wish to reproduce the experimental value of $g_A^{(0)}$ we need a suppression factor of about 1/4. This implies an unrealistically small value for $g_A \sim 5/3 \cdot 1/4 \sim 0.4$. The inclusion of the asymptotic one pion contribution does not help very much (See Fig. 1, as well as the discussions below). It increases g_A about by 50 % [17], but still g_A is only of the order of 0.6. The Skyrme model result $g_A^{(0)} = 0$ is obtained when the suppression factor vanishes but in this case g_A is also zero, which is inconsistent with the Skyrme model.

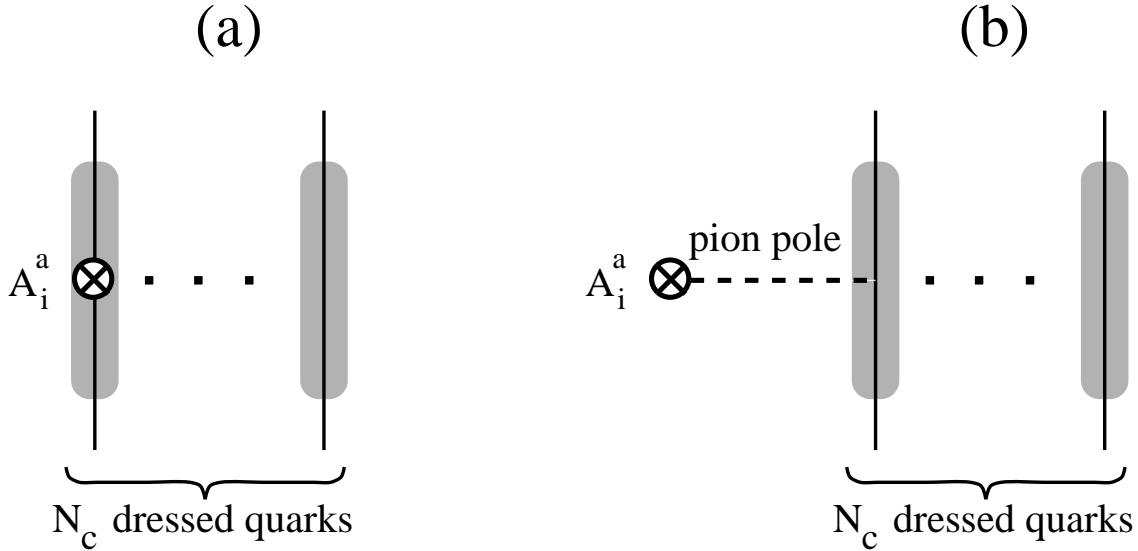


Figure 1: Two components of the isovector axial vector current A_i^a : (a) the current which directly couples to the quark line, and (b) the current which couples to the pion pole term. In both cases, the quarks are dressed by pion clouds as indicated by the hatched area. The number of the pions in the cloud around the quarks is related to N_π in (19).

Next we consider the pionic effect on the N_c quark state. This must be distinguished from the asymptotic one pion contribution from the tail region but is rather related to the finite range effect which generates the hedgehog structure through non-linear interactions in the large- N_c limit (Fig. 1). We consider an ansatz of the direct product of the N_c quarks and N_π pions. The $SU_\pi(4)$ algebra can be realized in terms of the same type of operators as the quark algebra. Since a pion carries spin one, the hedgehog form can be expressed as $\left[\frac{1}{\sqrt{2}}(\alpha_{u\downarrow}^\dagger - \alpha_{d\uparrow}^\dagger)\right]^2$. It is

convenient to lump the quarks and pions together [15] and write

$$|N_c; h_{q+\pi}\rangle = \sqrt{\frac{1}{(N_c + 2N_\pi)!}} \left[\frac{1}{\sqrt{2}}(\alpha_{u\downarrow}^\dagger - \alpha_{d\uparrow}^\dagger) \right]^{N_c + 2N_\pi} |0\rangle. \quad (19)$$

The meaning of the number of the pions is not very clear yet, but is related to how the nucleon spin is partitioned between quarks and pions. In a large- N_c baryon where the bound state of N_c quarks is treated in the Hartree approximation, the number of the pions in the baryon would be expected to be proportional to N_c , since in this approximation it is N_c times the number of the pions around a single quark, which one expects to be of order one. In the following argument, however, N_c and N_π are treated as independent parameters in order to understand the results in what follows.

Now the nucleon matrix elements for S_i and T_{ia} are computed in the same manner as before. Note that those operators act directly on the N_c quarks as illustrated in Fig. 1(a). The results are

$$\langle p \downarrow | S_3 | p \downarrow \rangle = -\frac{N_c}{N_c + 2N_\pi} \equiv -g_A^{(0)}, \quad (20)$$

$$\langle p \downarrow | T_{33} | p \downarrow \rangle = -\frac{N_c}{3} \left(1 + \frac{2}{N_c + 2N_\pi} \right) \equiv -g_A. \quad (21)$$

These results have interesting implications. The nucleon spin $g_A^{(0)}$ becomes less than unity for a finite number of pions $N_\pi \neq 0$, where a part of the nucleon spin is carried by the angular momentum of pions. When $N_\pi = 0$, Eq. (20) reduces to the quark model result, where the entire nucleon spin is carried by the quark spin, and when $N_\pi \rightarrow \infty$ we obtain the skyrmion result, where the nucleon spin is entirely carried by the pion cloud [18]. The same thing holds also for the second term of g_A in (21). In particular, the realization of the Skyrme model results in the limit $N_\pi \rightarrow \infty$ as shown here, rather than $N_c \rightarrow \infty$, is interesting. This result may be understood by interpreting the Skyrme soliton as a coherent superposition of infinitely many pions. At this point, it is interesting to recall the recent result by Dorey and Mattis, who have explicitly shown that the Skyrmion is an ultraviolet fixed point of a chiral bag model, where the coupling between the pion and the bare nucleon disappears [19]. This implies that the dynamics of the meson-baryon system is completely described by the pion alone.

Let us now try to make a rough estimate for $g_A^{(0)}$ and g_A . In the physical nucleon, both effects, the relativity through the $L = 1$ component and the pionic effects, must be considered. Some algebra shows that this can be done by multiplying both (20) and (21) by the reduction factor $|c_1|^2 - \frac{1}{3}|c_2|^2$. A reasonable estimate for this quantity is ~ 0.7 . For $N_c = 3$, g_A of (21)

becomes

$$g_A \sim 0.7 \left(1 + \frac{2}{3 + N_\pi} \right). \quad (22)$$

The pion contribution as depicted in Fig. 1(b) must still be added to this result. In the chiral limit this can be estimated to be about 50 % of the quark contribution of (22) [17], and so we get for the total g_A

$$g_A \sim 1.5 \times 0.7 \left(1 + \frac{2}{3 + N_\pi} \right) \sim \left(1 + \frac{2}{3 + 2N_\pi} \right).$$

The experimental value of $g_A \sim 1.3$ is then reproduced when $N_\pi \sim 2$. Note that the $1/N_c$ correction term in (23) is about 30 %, which is in good agreement with the previously quoted numbers in the chiral quark soliton model and in the chiral bag model [8, 9]. Using the same parameters, the nucleon spin $g_A^{(0)}$ comes out to be

$$g_A^{(0)} \sim 0.7 \frac{3}{3 + 2N_\pi} \sim 0.3.$$

It is indeed remarkable that such a simple algebraic method can be used to describe both g_A and $g_A^{(0)}$ simultaneously in good agreement with experiments.

6 Summary

In this note we have investigated algebraic models that contain a subgroup $SU(4)$ whose contractions reduce to the large- N_c algebra for QCD. We have explicitly constructed a realization which interpolates the skyrmion and the quark model results for $g_A^{(0)}$ and g_A . In order to find such a result we were inspired by models to include explicit pionic degrees of freedom. The agreement of the present calculations with experiments and with previous model calculations suggests that those quantities are strongly governed by the underlying algebraic structure, rather than their detailed dynamical details, as also implied by the large- N_c limit of QCD.

A detailed discussion of several algebraic models as phenomenological models compatible with the large- N_c behavior of QCD will be presented in Ref. [15].

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